# Transformations

## Objectives

Students will be able to identify and compare the three congruence transformations, apply the three congruence transformations to coordinates of the vertices of figures, identify and apply dilations, and apply transformations to real-world situations.

## Core Learning Goal

2.1.3 The student will use transformations to move figures, create designs and/or demonstrate geometric properties.

## Materials Needed

Worksheets, protractor, ruler, patty paper, Mira™

Optional – Dynamic geometry software

## Approximate Time

Four 45-minute lessons

## Additional Resources

National Council of Teachers of Mathematics (NCTM). *Navigating Through Geometry in Grades 6-8*, 2002, Chapter 3 -Transformations and Symmetry, pp. 43-58.

National Council of Teachers of Mathematics (NCTM). *Navigating Through Geometry in Grades 9-12*, 2001, Chapter 1 - Transforming Our World, pp. 9-26.

Dixon, Juli, *Movements in the Plane: Conjecturing about Properties of Transformations,* NCTM Math On-Line January 2003

# Transformations

## Lesson Plan – Translations

## Essential Questions

What are the similarities and differences between the images and pre-images generated by translations?

What is the relationship between the coordinates of the vertices of a figure and the coordinates of the vertices of the figure's image generated by translations?

How can translations be applied to real-world situations?

## Development of Ideas

Investigate translations using patty paper.

Worksheet: **Translations**

Answers:

2. a. The distances between the vertices of the pre-image and the image are the same.
   b. In a translation, the distance between every point in the pre-image and its corresponding point in the image are the same.
3. The lengths of the corresponding sides and the measures of the corresponding angles of the pre-image and the image are congruent.
4. A translation is an isometry because all of the side lengths and angle measures between the pre-image and its image are the same.

### Activity One (Optional)

Investigate and draw translations on a coordinate plane using dynamic geometry software.

Worksheet (Cabri): **Translations and Coordinate Geometry**

Answers:

2. a.-d. Answers will vary. The differences in the x- and y-coordinates should be constant in all responses.
   e. The vector for each point of the pre-image to the image will be the same.

## Development of Ideas (Continued)

Answers to Optional Activity One (Continued)

3. a.-b. Answers will vary. The differences in the x- and y-coordinates should be constant in all responses.
   c. The vector for each point of the pre-image to the image will be the same.
4. $(x + \Delta x, y + \Delta y)$
5. d. Add the x- and y-coordinates from $v_1$ to each vertex of $\Delta DEF$ for the first translation. Add the x- and y-coordinates from $v_2$ to each vertex of $\Delta DEF$ for the second translation.
   e. The single translation would be to use the sum of the two vectors, $v_1 + v_2$.
7. a. A (-5, 2), B (-2, 7), and C (0, 4)
   A′ (0, 0), B′ (3, 5), and C′ (5, 2)
   b. A″ (1, 3), B″ (4, 8), and C″ (6, 5)

## Closure

Summary questions:

If you pick up a paper clip and move it from your desk to the desk next to you, is this a translation? Why or why not?

Answer: It is a translation because the shape and the size of the paper clip stays the same, only it position changes.

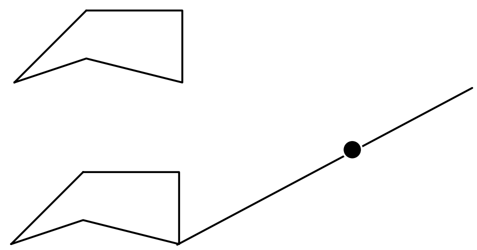
If you take a rubber band and stretch it tight, is this transformation of the rubber band an isometry? Why or why not?

Answer: This is not an isometry because the shape of the rubber band changes.

# Transformations

## Translations

Example



1. a. Draw a polygon on patty paper.
   This polygon is called the pre-image.

   b. Draw a line segment from one vertex toward the edge of the paper.

   c. Mark a point on your line segment.

   d. Trace the polygon and line on a second sheet of patty paper.

   e. Place one copy under the other aligning the corresponding points. Now slide the top picture so that the point on the line on the bottom paper and the vertex of the polygon on the top paper coincide, keeping the lines on top of each other.

   f. Trace both polygon figures on to the same sheet of patty paper.

   g. Using a third sheet of patty paper, mark the length of the segment from the vertex of the original polygon to the point marked on the line.

   h. Draw segments connecting corresponding vertices of the pre-image polygon and the image polygon. Compare the lengths of each segment with the marked length.

2. a. How do the distances between the vertices of the pre-image and the vertices of the image compare?

   b. Write a statement about the distance between any point and its image in a translation.

3. How do the lengths of the corresponding sides and the measures of corresponding angles of the pre-image and the image compare?

4. A translation is called a **rigid transformation** or an **isometry**. The word isometry can be broken into *iso* meaning the same, and *metry* meaning measure. Explain why a translation is an isometry.

# Transformations

## Translations and Coordinate Geometry (Optional)

1. a. Select **Show Axes** from the **Draw** toolbox to turn on the coordinate axes.
   b. Select **Define Grid** from the **Draw** toolbox then click on the axes to turn on the coordinate grid.
   c. Select Triangle tool from the **Lines** toolbox. Construct a triangle with vertices on grid points. Label the vertices A, B and C.
   d. Select **Equation & Coordinates** in the **Measure** toolbox, then click once on each vertex to show the coordinates of the vertex.
   e. Now translate the triangle.
      - Select **Vector** from the **Lines** toolbox. Draw a vector starting and ending on grid points.
      - Select the **Translation** tool from the **Transformations** toolbox.
      - Select the triangle (the object to translate).
      - Select the vector by which to translate the triangle.
   f. Label the corresponding image vertices A', B', and C' and display the coordinates of each point.

2. a. Record the coordinates of the vertices, the change in *x* and the change in *y* in the table below

| Vertex | Pre-image | Image | $\Delta x$ | $\Delta y$ |
|---|---|---|---|---|
| A | | | | |
| B | | | | |
| C | | | | |

   b. Display the coordinates of the endpoints of the vector.

   c. What is the change in *x* from the initial point of the vector to the endpoint of the vector?

   d. What is the change in *y* from the initial point of the vector to the endpoint of the vector?

   e. Describe the relationship between the vector and the image and pre-image.

3. a. On a new screen, repeat problems 1 and 2 above. Use the table below to record your results.

| Vertex | Pre-image | Image | $\Delta x$ | $\Delta y$ |
|---|---|---|---|---|
| A | | | | |
| B | | | | |
| C | | | | |

   b. $\Delta x$ = ______ $\Delta y$ = ______

   c. Describe the relationship between the vector and the image and pre-image.

## Translations and Coordinate Geometry (Optional) (Continued)

4. If an ordered pair represents one of the vertices of the pre-image $(x, y)$ and the translation vector has change in $x$ written as $\Delta x$, and a change in $y$ written as $\Delta y$, write the rule that gives the coordinates of the image.

$$(x, y) \rightarrow (\_\_\_\_\_\_\_\_\_\_, \_\_\_\_\_\_\_\_\_\_)$$

5. a. On a new screen, construct a triangle with its vertices at grid points. Label the vertices D, E, and F. Display the coordinates of each vertex.

   b. Construct a translation vector, $v_1$, starting and ending on grid points. Translate $\Delta DEF$ using this vector. Label the corresponding vertices of the image D', E' and F'. Display the coordinates of each vertex.

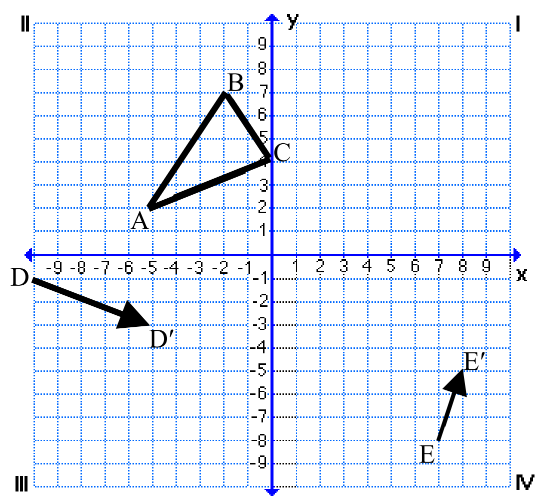
   c. Construct another translation vector, $v_2$, starting and ending on grid points. Translate $\Delta D'E'F'$ using this new vector. Label the corresponding vertices of the newest image D", E" and F".

   d. What is the rule for the first translation? What is the rule for the second translation?

   e. What single translation is the same as (equivalent to) translating by $v_1$ and then by $v_2$?

6. Applying one translation to a figure and then applying a second translation to its image is called a **composition** of translations.

7. a. On the grid below, what are the coordinates of A, B and C? Now translate the triangle using the vector $\overrightarrow{DD'}$. What are the coordinates of the image triangle A'B'C'?




   b. Now translate the image triangle by vector $\overrightarrow{EE'}$. What are the coordinates of A", B" and C"?

# Transformations

## Lesson Plan – Line Symmetry and Reflections

## Essential Questions

What are the similarities and differences between the images and pre-images generated by reflections?

What is the relationship between the coordinates of the vertices of a figure and the coordinates of the vertices of the figure’s image generated by reflections?

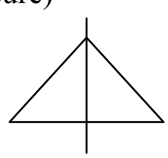
How can reflections be applied to real-world situations?

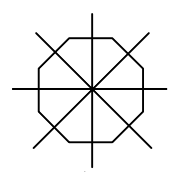
## Warm-Up/Opening Activity

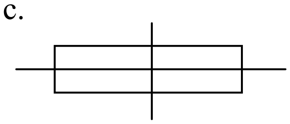
Investigate reflectional symmetry.

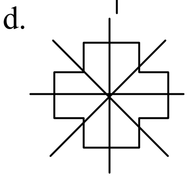
Worksheet: **Line Symmetry**

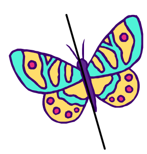
Answers:

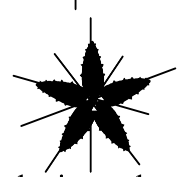
2. Answers will vary depending on student designs.
3. Using the term reflectional symmetry makes sense because one side of the line of symmetry is a reflection of the other side of the line.
4. A, B, C, D, E, H, I, K, M, O, T, U, V, W, X, Y (check font to be sure)
5. a. 

   b. 

   c. 

   d. 

   e. 

   f. 
6. a. Answers will vary (example: isosceles triangle).
   b. Answers will vary (example: rectangle).
   c. Answers will vary (example: equilateral triangle).
   d. Answers will vary (example: square).
7. No, for example, a rectangle’s lines of symmetry pass through the midpoints of each side.

# Transformations

## Development of Ideas

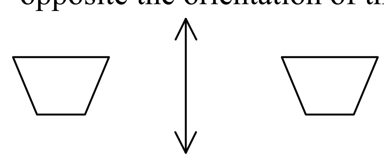
### Activity Two

Investigate and apply reflections using a Mira™ and patty paper.

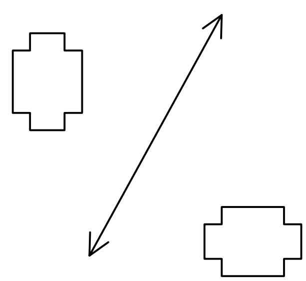
Worksheet: **Reflections**

Answers:

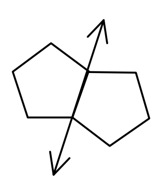
1. e. No, the segments are not all the same length.
   f. The angle measures are congruent (180º).
   g. The image is congruent to the pre-image, therefore, the reflection is an isometry.
   h. The lengths of the two parts of each segment are equal.
   i. The line of reflection is the <u>perpendicular bisector</u> of every segment connecting a point of the pre-image and its image.
2. a.-b. Answers will vary.
   c. The size of the pre-image and image are the same, the distance from the line of reflection is the same, and the orientation of the image is opposite the orientation of the pre-image.
3. a.



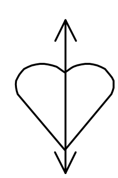
b.



c.



d.

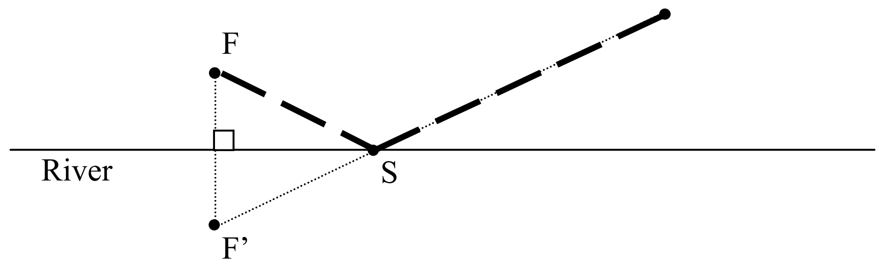


B

## Development of Ideas (Continued)

Answers to Activity Two (Continued)

4.




### Activity Three (Optional)

Investigate and draw reflections on a coordinate plane.

Worksheet (Cabri): **Reflections on a Coordinate Plane**

Answers:

2. a. Answers will vary.
   b. The x-coordinates of A to A′ are opposites of one another and the y-coordinates of these two points are the same.
   c. The other points have the same characteristics as the relationship between A and A′.
3. a. Answers will vary.
   b. The corresponding vertices have the same x-coordinates and the y-coordinates are opposites.
4. a. y = x
   b. Answers will vary.
   c. Answers will vary.
   d. The x- and y-coordinates interchange with one another, (x, y) → (y, x).
5. a. Coordinates of A (-4, 3) and A′ (-4, -3)
      (x, y) → (x, -y).
   b. Coordinates of B (3, 5) and B′ (-3, 5)
      (x, y) → (-x, y).
   c. Coordinates of A (-4, 3) and A′ (3, -4)
      (x, y) → (y, x).

# Transformations

## Closure

Summary questions

Compare translations and reflections. What is the same? What is different?

Answer: The size and the shape of the pre-image and image are the same (because they are isometries) but the orientation is different in reflections compared to translations (pre-image and image have the same orientation).

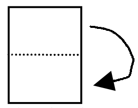
Janet constructed the perpendicular from each of the vertices of a triangle to a line. How will this help her to find the image of the triangle reflected over the line?

Answer: Since Janet has constructed the perpendicular to a line from each vertex, she can use each constructed line to help create a reflection. Janet can measure the distance from each vertex to the line, copy that distance to the other side of the line and mark a new vertex. This new vertex will be the reflection of the original vertex. Repeating this for each vertex and connecting the vertices, Janet will complete a reflection of the original triangle.

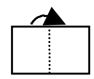
# Transformations

## Line Symmetry

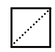
1. a. Using a blank sheet of square paper, carefully fold the paper in half.



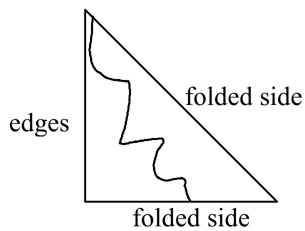
b. Now fold the paper in half again.



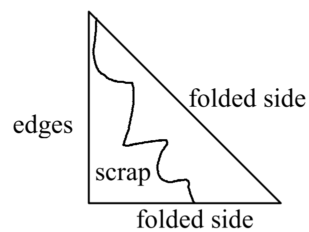
c. Fold once more diagonally.



d. Draw a design on one side of the paper.
For example:




e. Carefully cut along the drawn line.
Throw away the scrap.




f. Sketch what your design will look like if it is unfolded completely.

g. Unfold your design and compare it to the sketch.

2. A design has **line symmetry** if the design can be folded along one line and one side is the mirror image of the other. The line is called the **line of symmetry**.

How many lines of symmetry does your design have? Check your answer by folding your design along each line of symmetry.

3. Line symmetry is sometimes called **reflectional symmetry**. Using what you know about line symmetry, why does the word reflectional make sense?

4. Which letters of the alphabet have reflectional symmetry?

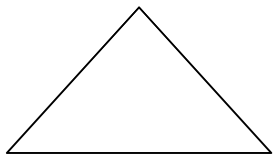
A B C D E F G H I J K L M N O P Q R S T U V W X Y Z

# Transformations

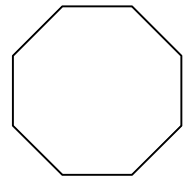
## Line Symmetry (Continued)

5. For each of the following shapes, draw the line(s) of symmetry.

a.

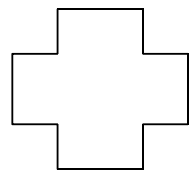


b.



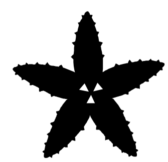
c.

d.



e.

f.



6. a. Draw a polygon that has one line of symmetry. A figure with only one line of symmetry is said to have **bilateral symmetry**.

   b. Draw a polygon that has two lines of symmetry.

   c. Draw a polygon that has three lines of symmetry.

   d. Draw a polygon that has four lines of symmetry.

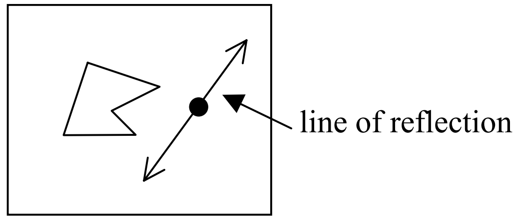
7. Does a line of symmetry always pass through the vertices of the figure? Use mathematics to justify your answer.

# Transformations

## Reflections

1. a. Draw a simple polygon on a sheet of patty paper. Then draw a line on the patty paper outside the figure and place a point on the line.

Example




b. On a second sheet of patty paper, trace the polygon, the line of reflection and the point.

c. Flip the second piece of patty paper over and place it under the first so that the lines and the point coincide. Now trace the image onto the first patty paper.

d. Connect a point in the pre-image with its corresponding image point. Repeat this for two other points on the original polygon.

e. Compare the lengths of each segment connecting a point with its image. Are the segments all the same length?

f. Measure the angle formed by the line of reflection and each of the segments connecting a point and its image. What are the measures of these angles? How do the measures compare?

g. Fold the patty paper along the line of reflection. What can you say about the size of the image compared to the pre-image? Is a reflection an isometry?

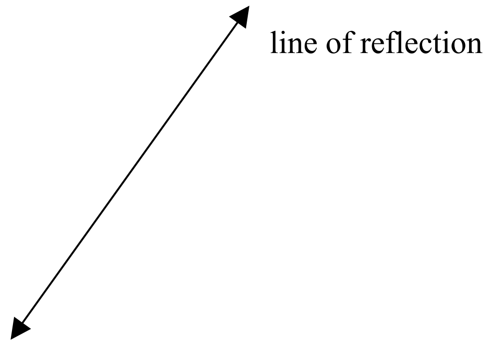
h. The line of reflection divides each segment connecting a point and its image into two parts. Compare the lengths of the two parts of each segment.

i. Complete the following statement:

The line of reflection is the _______________ ______________ of every segment connecting a point of the pre-image and its image.

## Reflections (Continued)

2. a. Draw a simple polygon on one side of the line below. Label the vertices, A, B, C, etc.

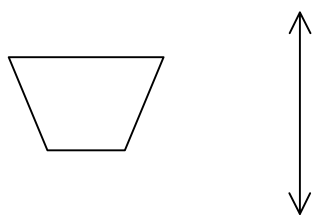



b. Place a Mira™ on the line of reflection and mark the vertices of the image. Then complete the image and label the corresponding vertices, A', B', C', etc.

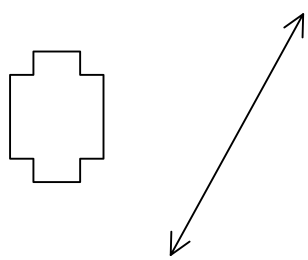
c. Write a statement comparing the size, position relative to the line of reflection, and orientation of the figure and its image.

3. Use your Mira™ to construct the image of each figure over the given line of reflection.

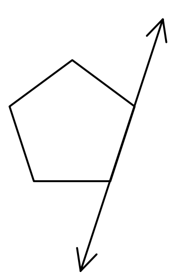
a.



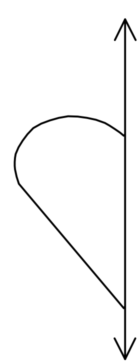
b.



c.



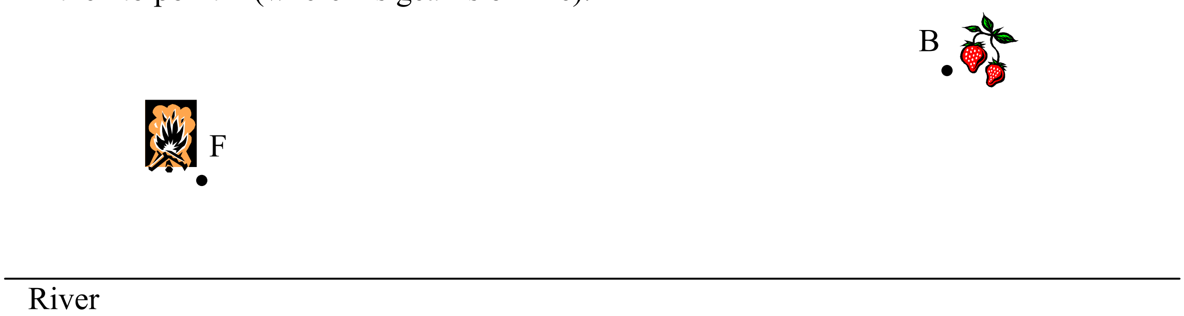
d.



# Transformations

## Reflections (Continued)

4. A reflection can be used to find the shortest path. Suppose that Bob is camping along the river. He sets up his tent and builds a fire at point F. He walks through the woods picking berries for his dinner. When he looks back at the camp, he notices that a spark has set his gear on fire. He needs to get water from the river and put out the fire as soon as possible. He needs the shortest path from point B (the berries) to the river and then to point F (where his gear is on fire).




To find the shortest path:

a. Reflect point F over the line that is the river's edge.

b. Draw $\overline{F'B}$.

c. Label the intersection of the river edge and $\overline{F'B}$ as point S.

d. Bob should run from B to S, pick up the water in his berry bucket, and then run from S to F, to put out the fire. Draw his path.

# Transformations

## Reflections on a Coordinate Plane (Optional)

1. a. Select **Show Axes** from the **Draw** toolbox to turn on the coordinate axes.
   b. Select **Define Grid** from the **Draw** toolbox then click on the axes to turn on the coordinate grid.
   c. Select Triangle tool from the **Lines** toolbox. Construct a triangle with vertices on grid points. Label the vertices A, B and C.
   d. Select **Equation & Coordinates** in the **Measure** toolbox, then click once on each vertex to show the coordinates of the vertex.
   e. Now reflect the triangle.
      - Select the **Reflection** tool from the **Transformations** toolbox.
      - Select the triangle (the object to reflect).
      - Select the y-axis as the line over which to reflect the triangle.
   f. Label the corresponding image vertices A', B', and C' and display the coordinates of each point.

2. a. Record the coordinates of the vertices when the pre-image is reflected over the y-axis.

| Original Vertex | Pre-image | Image |
|---|---|---|
| A | | |
| B | | |
| C | | |

   b. How are the coordinates of A and A' related?

   c. How are the other corresponding vertices related?

3. a. Reflect the original triangle over the x-axis. Label the vertices as A", B" and C". Record the coordinates in the table below.

| Original Vertex | Pre-image | Image |
|---|---|---|
| A | | |
| B | | |
| C | | |

   b. How are the corresponding vertices related (A to A", B to B", and C to C")?

4. a. Construct a line through the origin and through the point (1, 1). What is the equation of this line?

   b. Reflect the original triangle over the line in part a. Label the coordinates of the vertices.

   c. Record the coordinates in the table on the next page.

## Reflections on a Coordinate Plane (Optional) (Continued)

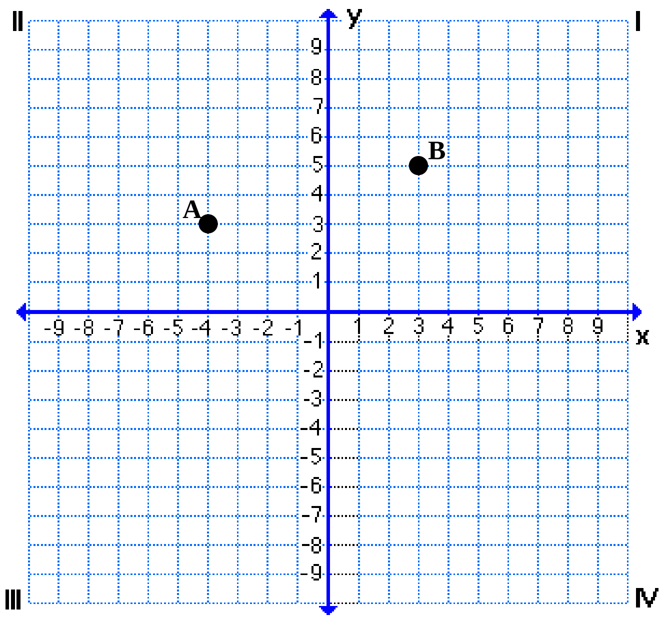
c. Complete the table.

| Original Vertex | Pre-image | Image |
|---|---|---|
| A | | |
| B | | |
| C | | |

d. How are the corresponding vertices related?

5. To summarize the ideas in problems 1- 4, fill in the work below:

a. Reflect point A over the ***x***-axis.




Coordinates of A ( , )
Coordinates of A' ( , )

For any point $(x, y)$
To reflect over the $x$-axis
$(x, y) \to (\quad , \quad)$

b. Reflect point B over the ***y***-axis.

Coordinates of B ( , )
Coordinates of B' ( , )

For any point $(x, y)$
To reflect over the $y$-axis
$(x, y) \to (\quad , \quad)$

c. Reflect point A over the line $\boldsymbol{y = x}$.

Coordinates of A ( , )
Coordinates of A' ( , )

For any point $(x, y)$
To reflect over the line $y = x$
$(x, y) \to (\quad , \quad)$

# Transformations

## Lesson Plan – Rotational Symmetry and Rotations

## Essential Questions

What are the similarities and differences between the images and pre-images generated by rotations?

What is the relationship between the coordinates of the vertices of a figure and the coordinates of the vertices of the figure's image generated by rotations?

How can rotations be applied to real-world situations?

## Warm-Up/Opening Activity

Investigate rotational symmetry

Worksheet: **Rotational Symmetry**

Answers:

1. a. There are 180º in a half-turn.
   b. There are 90º in a quarter-turn.
3. a. You can rotate the equilateral triangle three times so that the sides and point line up.
   b. There are 120º in each turn.
   c. You can turn a 5-fold symmetry 5 times.
4. a. 4 b. 2 c. 6
5. a. 90º b. 180º c. 60º
6. b. The figure needs to be turned 180º to line up again.
7. The number of degrees of each turning is 360 degrees divided by the number of times a figure can be rotated.
8. The letters H, I, O, S, X, and Z have rotational symmetry.
9. The cards 2 of clubs, 8 of diamonds, 5 of diamonds, and jack of spades have rotational symmetry (the ace of clubs does not).

## Development of Ideas

### Activity Four

Investigate and apply rotations using patty paper.

Worksheet: **Rotations**

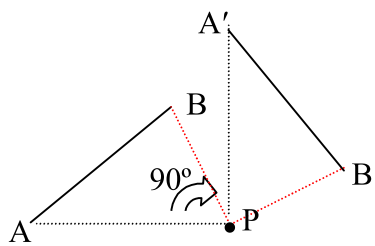
## Development of Ideas (Continued)

Answers to Activity Four

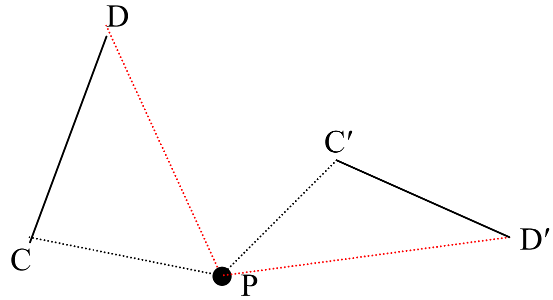
1. h. When a figure is rotated around a point, the shape is *not changed*, the orientation of the image in *changed* and the distance between points and their images are *the same* but the angle formed by a point and its image with the center of rotation is always *the same.*

2. a. The image of point A is the point E.
   b. The pre-image of the point G is the point C.
   c. The image of $\overline{CD}$ is $\overline{GF}$.

3.




4.




5. The line segment should be rotated in the other direction and the angle should be constructed in the other direction.

6. a. The point M rotated 120° counter clockwise is the point N.
   b. The point M rotated 120° clockwise is the point O.

# Transformations

## Development of Ideas (Continued)

## Closure

Summary questions

Compare translations, reflections and rotations. What properties stay the same? What properties are different? Make sure to consider segment lengths, angle measures, perimeter and volume in your answer.

Answer: Since translations, reflections, and rotations are isometries, all of the transformations produce congruent images, meaning the segment length, angle measure, perimeter, and volumes of pre-images and images are congruent. The orientation of the figures may be different.

# Transformations

## Rotational Symmetry

1. a. How many degrees are there in a half-turn?

   b. How many degrees are there in a quarter-turn?

2. a. Construct a large equilateral triangle on a piece of patty paper.

   b. Fold the paper to construct the medians of two sides and mark the centroid of the triangle.

   c. Mark a reference point outside the triangle on the patty paper.

   d. Now trace the triangle, centroid, and the reference point outside the triangle onto a second piece of patty paper.

   e. Place the copy on top of the original so that they match up all marks.

   f. Hold the two copies together by pressing down with the point of a pencil on the centroid. Carefully rotate the triangle until the sides match up again.

3. Any figure that can be turned around a point by less than a full circle and match the original has **rotational symmetry** or **turn symmetry**.

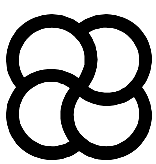
   a. How many times can you rotate the triangle until not only the sides but also the reference point outside the triangle match up?

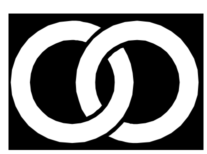
   b. How many degrees was each rotation of the triangle?

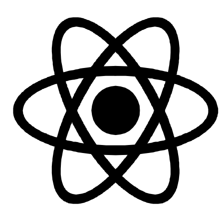
   c. We say that the equilateral triangle has **3-fold symmetry** since the triangle can be turned around the center three times before it is back to its original position.

   d. How many times could you turn a figure that has 5-fold symmetry?

4. If you turn each figure up to 360° about the center point, how many times will the figure match the original?

   a. 

   b. 

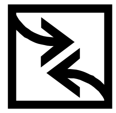
   c. 

## Transformations

### Rotational Symmetry (Continued)

5. How many degrees were there in each turn for each of the figures in 4?

6. a. 2-fold symmetry is often called **point symmetry**. The figure below has point symmetry.



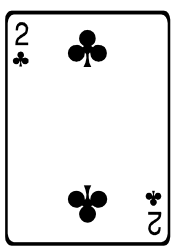
   b. How many degrees did the figure need to be turned to match up again?

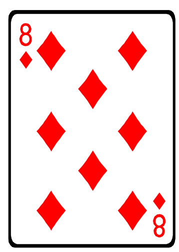
7. How does the number of times a figure can be rotated relate to the number of degrees it is turned each time?

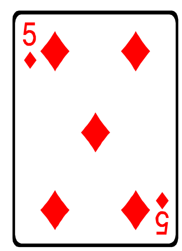
8. Which letters of the alphabet have rotational symmetry?

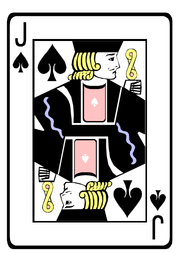
A B C D E F G H I J K L M N O P Q R S T U V W X Y Z

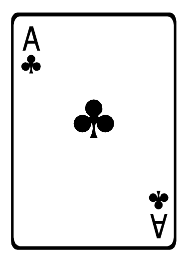
9. Which of these cards has rotational symmetry?











# Transformations

## Rotations

1. a. On a piece of patty paper, draw a small polygon. Label vertices X, Y, etc. Make sure the figure is drawn toward the side of the patty paper.

   b. Mark a point P on the paper. Draw an acute angle APB with vertex at P. Determine the measure of $\angle APB$.

   c. On a second piece of patty paper, trace the polygon and point P. Also trace $\overrightarrow{PA}$.

   d. Stack the papers and align the polygon and point P. Holding point P aligned, turn the bottom paper until its $\overrightarrow{PA}$ is aligned with $\overrightarrow{PB}$ on the top paper. Trace the image onto the first piece of patty paper.

   e. Locate the vertex X of the original polygon and label its corresponding vertex on the image polygon as X'. Draw $\overline{XP}$ and $\overline{X'P}$. Measure the angle formed. Measure the length of each segment.

   f. Locate the vertex Y of the original polygon and label its corresponding vertex on the image polygon as Y'. Draw $\overline{YP}$ and $\overline{Y'P}$. Measure the angle formed. Measure the length of each segment.

   g. Measure XX' and YY'. Are the distances between points on the original polygon and their corresponding image points always the same?

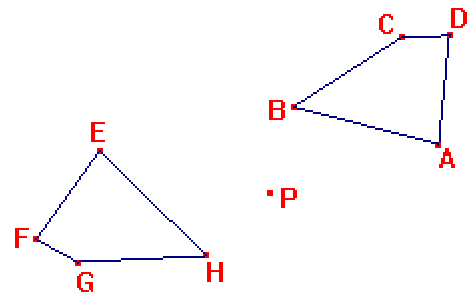
   h. Correctly complete the following statement concerning rotations by circling the correct word in each pair of italicized words that makes a true statement about rotation.

**Rotation:**
When a figure is rotated around a point, the shape is *changed/not changed*, the orientation of the image in *changed/not changed* and the distance between points and their images are *the same/different* but the angle formed by a point and its image with the center of rotation is always *the same/different*.

# Transformations

## Rotations (Continued)

2. The polygon EFGH is the image of polygon ADCB under a counterclockwise rotation of $150^\circ$ around point P.

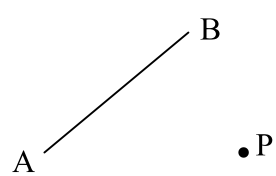



   a. Name the image of point A.

   b. Name the pre-image of G.

   c. What is the image of $\overline{CD}$?

3. To construct the image of the segment AB after a rotation of $90^\circ$ clockwise around point P follow the steps below.




- Draw $\overline{AP}$.
- Draw an $90^\circ$ angle with vertex at P, side $\overline{AP}$, and the other side above and to the right of $\overline{AP}$.
- Copy the length of $\overline{AP}$ on to the other side of the $90^\circ$ angle you drew. Mark point A'.

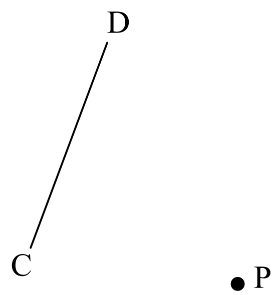
*Use a different color pencil for the next steps.*

- Draw $\overline{BP}$.
- Draw an $90^\circ$ angle with vertex at P, side $\overline{BP}$, and the other side above and to the right of $\overline{BP}$.
- Copy the length of $\overline{BP}$ on to the other side of the $90^\circ$ angle you drew. Mark point B'.
- Finally draw segment A'B'.

# Transformations

## Rotations (Continued)

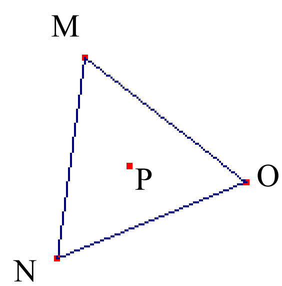
4. Construct the image of the segment $\overline{CD}$ after a rotation of $120^\circ$ clockwise around point P.




5. How should the angle be drawn if the instruction was to rotate the line segment counterclockwise?

6. Sometimes the center of rotation is inside the figure. Using $\Delta MNO$ below, draw a segment from the center of rotation, point P, to a vertex. Copy the triangle and the segment to patty paper. Draw the angle of rotation, $120^\circ$ counterclockwise, on the triangle below. Then place the patty paper on the figure, aligning point P. Rotate the patty paper until the segment on the copy aligns with the other side of the angle of rotation.

    a. Where does point M end up if you rotate the figure $120^\circ$ counterclockwise about point P?




    b. Where does point M end up if you rotate the figure $120^\circ$ clockwise about point P?

# Transformations

## Lesson Plan – Reflections over Two Lines and Dilations

## Essential Questions

What are the similarities and differences between the images and pre-images generated by each of the rigid transformations?

What is the relationship between the coordinates of the vertices of a figure and the coordinates of the vertices of the figure's image generated by each of the rigid transformations?

How can transformations be applied to real-world situations?

## Warm-Up/Opening Activity

Investigate reflections over parallel and intersecting lines.

Worksheet: **Reflections over Two Lines**

Answers:

1. a.-c. Answers will vary.
   d. The lengths of the segments $\overline{AA''}, \overline{BB''}$, and $\overline{CC''}$ are all the same (congruent).
   e. The size of the figures is the same, the positions are on opposite sides of the lines of reflection and the same distance from the line over which it was reflected, and the orientation of the pre-image to the second reflection is the same.
2. a.-c. Answers will vary.
   d. The lengths of the segments $\overline{AA''}, \overline{BB''}$, and $\overline{CC''}$ are all the same (congruent).
   e. The size of the figures is the same, the positions are on opposite sides of the line of reflection and the same distance from the line over which it was reflected, and the orientation of the pre-image to the second reflection is a rotation about the point P.

## Development of Ideas

### Activity Five

Investigate dilations using measurement

Worksheet: **Dilations using Measurement**

## Development of Ideas (Continued)

Answers to Activity Five:

1. i. $\Delta ABC$ and $\Delta DEF$ are similar. The corresponding angles are congruent and the ratio of the lengths of the corresponding sides is the same, 1:2.

2. i. $\Delta ABC$ and $\Delta XYZ$ are similar. The corresponding angles are congruent and the ratio of the lengths of the corresponding sides is the same, 2:1.

3. a. Dilations only preserve angle measure.
   b. A dilation is not an isometry. An isometry preserves both angle measure and length of sides, and a dilation only preserves angle measure.

4. a. The center of dilation in problem 2 is the point Q.
   b. $\Delta XYZ$ is the image triangle.

5. g. The two quadrilaterals are similar. The corresponding angles are congruent and the ratio of the lengths of the corresponding sides is the same.
   h. The scale factor is $\frac{1}{2}$.
   i. Point T is the center of dilation.

Activity

5. Answers will vary. The ratio of the sides of the colored paper to the lengths of string should be 3:1.
6. The scale factor is $\frac{1}{3}$.
7. The center of dilation is the location of the knot of string tied to the chair.
9. The two rectangles are similar because they have congruent angle measures and the ratio of the lengths of the corresponding sides is the same.
10. The center of dilation is still the point where the strings come together.
11. The string would have to go through the other side of the wall and the points would be twice the distance from the chair to the colored paper on the wall.

## Transformations

### Closure

Summarize transformations using the worksheet **Transformations**

Answers:

1. Reflection with a pre-image, image, and line of reflection.
2. Rotation with a pre-image, image, and angle of rotation.
3. Translation with a pre-image, image, and translation vector.
4. Samples:

   A translation takes the original figure, or pre-image, and slides it a given distance and direction. This distance and direction is represented by a translation vector. The resulting figure, or image, is a rigid transformation, or isometry.

   A reflection takes an original figure, or pre-image, and flips it over a line of reflection. The resulting figure, or image, is the same shape and size as the pre-image, making it a rigid transformation. The orientation of the image is the reverse of the pre-image.

   A rotation takes an original figure, or pre-image, and turns it about a point of rotation through a particular angle of rotation, including the direction (clockwise or counter clockwise). The resulting figure, or image, is the same shape and size as the pre-image. This makes it a rigid transformation.

# Transformations

## Reflections over Two Lines

1. a. Lines *l* and *k* below are a pair of parallel lines. On the left of line *l*, draw a small polygon near line *l*. Label the vertices A, B, C, etc.


b. Use the Mira™ to reflect the figure over line *l*. Label the corresponding vertices of the image A', B', C', etc.

c. Now use the Mira™ to reflect the image over line *k*. Label the new image corresponding vertices as A", B", C", etc.

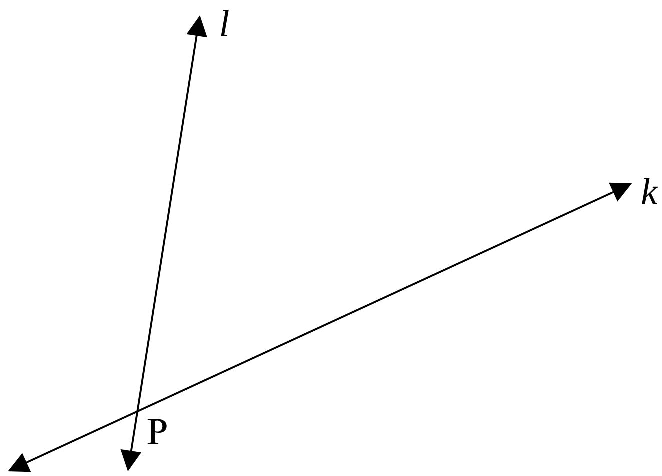
d. Draw the segment connecting vertices A and A". Draw the segment connecting B and B". Draw the segment connecting another pair of corresponding vertices. Compare the lengths of each segment drawn.

e. Write a statement comparing the size of the figures, positions relative to the lines of reflection, and orientation of the figure and its second image.

# Transformations

## Reflections over Two Lines (Continued)

2. a. Lines *l* and *k* intersect at point P. On the left of line *l*, draw a small polygon near line *l*. Label the vertices A, B, C, etc.




b. Use the Mira™ to reflect the figure over line *l*. Label the corresponding vertices of the image A', B', C', etc.

c. Now use the Mira™ to reflect the image over line *k*. Label the new image corresponding vertices as A", B", C", etc.

d. Draw the segment connecting vertices A and A". Draw the segment connecting B and B". Draw the segment connecting another pair of corresponding vertices. Compare the lengths of each segment drawn.

e. Write a statement comparing the size of the figures, positions relative to the lines of reflection, and orientation of the figure and its second image.

# Transformations

## Dilations using Measurement

1. a. On a blank piece of paper place a point P in the upper right corner.

   b. Draw any triangle ABC below and to the left of the point P. Make sure the triangle is fairly small. Determine the lengths of the sides and the measures of the three angles in $\Delta ABC$.

   c. Draw three rays $\overrightarrow{PA}$, $\overrightarrow{PB}$, and $\overrightarrow{PC}$.

   d. Measure $\overline{PA}$, $\overline{PB}$, and $\overline{PC}$.

   e. Mark a point D on $\overrightarrow{PA}$ so that 2PA = PD.

   f. Mark point E on $\overrightarrow{PB}$ so that 2PB = PE.

   g. Mark point F on $\overrightarrow{PC}$ so that 2PC = PF.

   h. Connect the points D, E and F to form triangle DEF. Determine the lengths of the sides and the measures of the three angles in $\Delta DEF$.

   i. Compare triangles ABC and triangle DEF. Write a statement about the two triangles.

2. a. On a second blank piece of paper place a point Q in the upper left corner.

   b. Draw any triangle ABC below and to the right of the point Q. Make sure the triangle is fairly large and toward the bottom of the page. Determine the lengths of the sides and the measures of the three angles in $\Delta ABC$.

   c. Draw three rays $\overrightarrow{QA}$, $\overrightarrow{QB}$, and $\overrightarrow{QC}$.

   d. Measure $\overline{QA}$, $\overline{QB}$, and $\overline{QC}$.

   e. Mark a point X on $\overrightarrow{QA}$ so that $\frac{1}{2}QA = QX$.

   f. Mark point Y on $\overrightarrow{QB}$ so that $\frac{1}{2}QB = QY$.

   g. Mark point Z on $\overrightarrow{QC}$ so that $\frac{1}{2}QC = QZ$.

   h. Connect the points X, Y and Z to form triangle XYZ. Determine the lengths of the sides and the measures of the three angles in $\Delta XYZ$.

   i. Compare triangles ABC and triangle XYZ. Write a statement about the two triangles.

3. In problems 1 and 2 you created a transformation called a **dilation**.

   a. Does a dilation preserve lengths of sides and measures of angles?

   b. Is a dilation an isometry? Use mathematics to justify your answer.

4. In problem 1 point P was called the center of dilation and triangle DEF was the image of triangle ABC under a dilation.

   a. Name the center of dilation in problem 2.

   b. Name the image triangle under the dilation in problem 2.

## Dilations using Measurement (Continued)

5. a. Place a point T in the center of a third piece of paper.
   b. Draw a large quadrilateral ABCD so that point T is in the interior of ABCD.
   c. Draw rays $\overrightarrow{TA}$, $\overrightarrow{TB}$, $\overrightarrow{TC}$, and $\overrightarrow{TD}$.
   d. Measure the lengths of $\overline{TA}, \overline{TB}, \overline{TC}$, and $\overline{TD}$.
   e. Mark points K, L, M, and N on $\overrightarrow{TA}$, $\overrightarrow{TB}$, $\overrightarrow{TC}$, and $\overrightarrow{TD}$ such that each point is the midpoint of $\overline{TA}$, $\overline{TB}$, $\overline{TC}$, and $\overline{TD}$ respectively.
   f. Draw the new quadrilateral KLMN.
   g. Are the two quadrilaterals are similar? Use mathematics to justify your answer.
   h. A scale factor for a dilation is the factor by which a figure is enlarged or reduced. What is the scale factor for the dilation that starts with quadrilateral ABCD and ends with quadrilateral KLMN?
   i. What is the center of the dilation for this transformation?

**Activity**

Materials needed: piece of colored paper, string, tape, meter sticks, wall and chair.

1. Tape the piece of colored paper to the wall. Label the corners A, B, C and D.
2. Cut four pieces of string each 3 meters long and tape one piece to each corner of the piece of colored paper.
3. Gently pull all four pieces of string together and tie the group to the back of a chair placed in front of the wall and colored paper. Make sure each piece of string is tight.
4. Measure 1 meter from the chair along each string and mark the point. With a piece of string, connect the points marked to form a rectangle.
5. What are the lengths of each side of the pre-image rectangle (colored paper) and each side of the image rectangle (string rectangle)?

6. What is the scale factor for this dilation?

7. What is the center of dilation?

8. Now mark a point on each of the strings so that the scale factor is $\frac{1}{2}$ and make a third rectangle.
9. Compare the third rectangle and the colored piece of paper. Are they similar? Use mathematics to justify your answer.
10. What is the center of dilation?

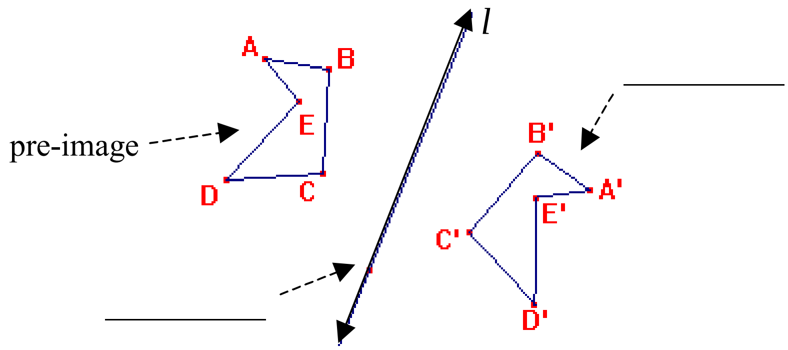
11. Where would you mark the string to have a scale factor of 2?

**Transformations**

Use the word bank at the bottom of the page to fill in each blank.

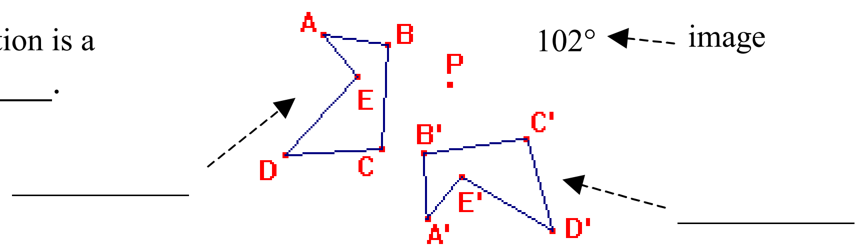
1. This transformation is a

   ________________.



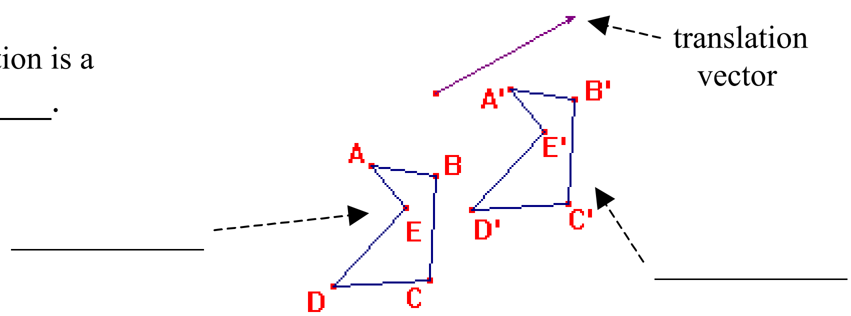

2. This transformation is a

   ________________.




3. This transformation is a

   ________________.




4. Write a careful description of translation, reflection, and rotation. Use the Word Bank to help complete your descriptions.

| **WORD BANK** | |
|---|---|
| image | preimage or original |
| rotation | reflection |
| translation | translation vector |
| direction | distance |
| angle of rotation | center of rotation |
| line of reflection | rigid transformation |